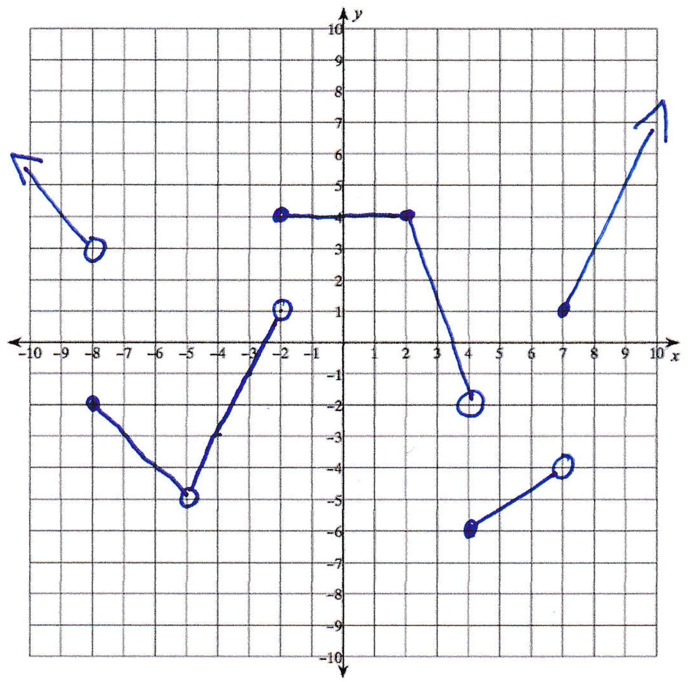
Bergen Catholic AP Calculus Summer Packet

Instructions:

- Students entering AP Calculus AB in September, must complete questions #1-60
  - There will be three Review Tests at the Start of the School year based on this summer packet
    - Unit 1 (#1-20): 1st Week
    - Unit 2 (#21-40): 2nd Week
    - Unit 3 (#41-60): 3rd Week
  - Make sure you are taking the Summer Packet questions seriously as you prepare for these Review Tests!
- Students entering AP Calculus BC in September, must complete questions #1-100
  - There will be two Review Tests at the Start of the School year based on this summer packet
    - Units 1-3 (#1-60): 1st Week
    - Units 4-5: 2nd Week
  - Make sure you are taking the Summer Packet questions seriously as you prepare for these Review Tests!

Part I: Use the graph below to evaluate each of the following limits



1) $\lim_{x\to -8^+} f(x)$

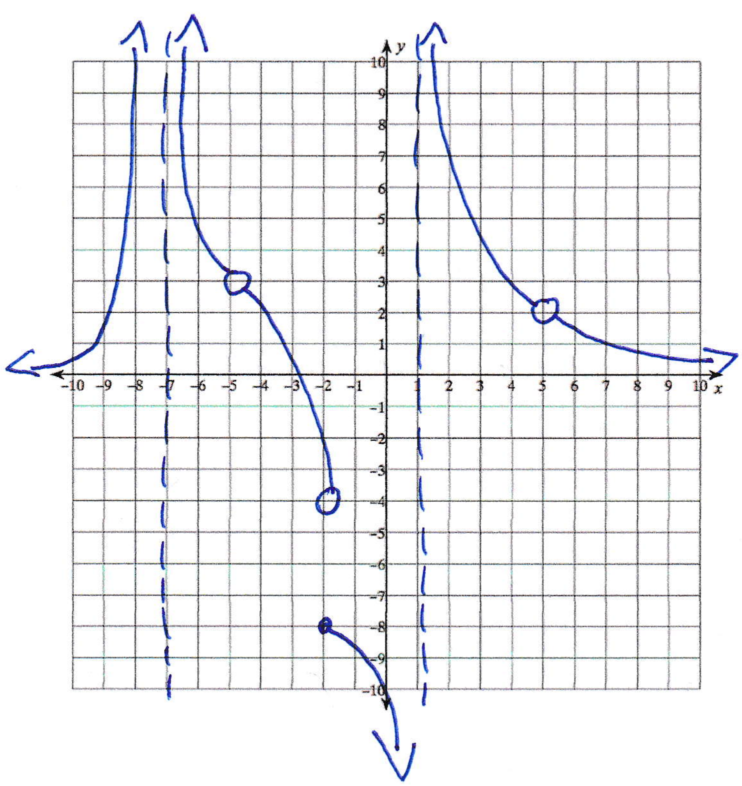
2) $\lim_{x\to -5} f(x)$

3) $\lim_{x\to -3} f(x)$

4) $\lim_{x\to 4^-} f(x)$

5) $\lim_{x\to 7} f(x)$

Part II: Use the graph below to determine the discontinuities of the function. Identify each discontinuity as removable vs. nonremovable and classify the nonremovable discontinuities by type.



6) Removable:

7) Nonremovable Infinite:

8) Nonremovable Jump:

Part III: Evaluate each limit algebraically. Watch out for 0/0 limits and limits with infinity.

9) $\lim_{x\to 2} \frac{x^2-4}{x^2-8x-12}$

12) $\lim_{x\to \frac{3\pi}{4}} \csc x$

15) $\lim_{x\to -\infty} \frac{9x^2+5x-4}{6x^2-2x+3}$

10) $\lim_{x\to -3} \frac{\sqrt{x+7}-2}{x+3}$

13) $\lim_{x\to -\frac{11\pi}{6}} \sec x$

16) $\lim_{x\to \infty} 4e^{\frac{5}{x}} + 7$

11) $\lim_{x\to 8} \frac{x^2-10x+16}{64-x^2}$

14) $\lim_{x\to \infty} \frac{4x^2-2x+10}{-6x-2}$

17) $\lim_{x\to -\infty} -3e^{2x} + 1$

Part IV: Use the piecewise function below to evaluate the given limit

$$f(x) = \begin{cases} -x^2 - 3x, & x < -2 \\ 3x + 8, & -2 \leq x < 4 \\ \sqrt{x-4}, & x \geq 4 \end{cases}$$

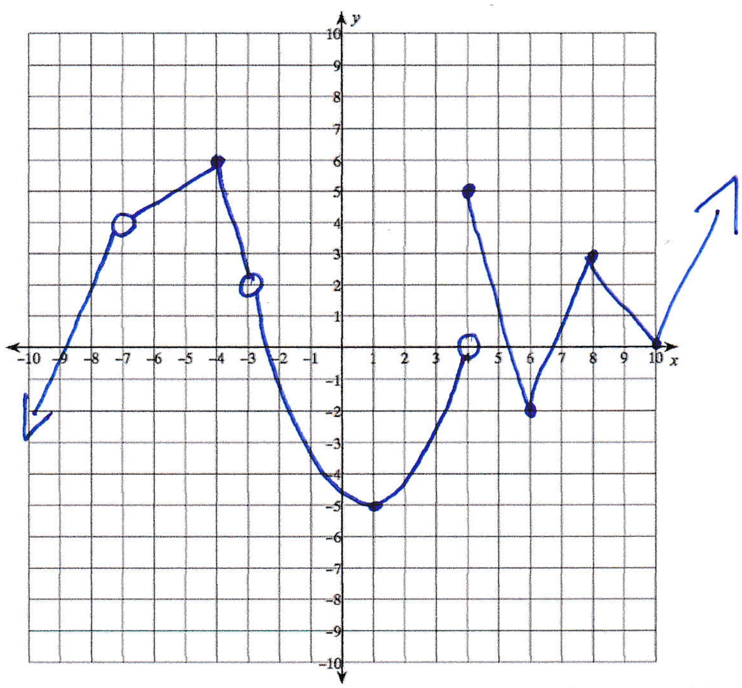
18) $\lim_{x\to -2} f(x)$

19) $\lim_{x\to 4^-} f(x)$

20) $\lim_{x\to 8^+} f(x)$

Part V: Use the graph below to identify all locations where the function is continuous but not differentiable.

21) Continuous but not differentiable at ________________________________



Part VI: Use the Power Rule to find the derivative of the given functions

22) $f(x) = 7x^8 - 4x^3 + 9x$

24) $f(x) = 4\sqrt[5]{x^9} - 3\sqrt[7]{x^{12}} + 8\sqrt[9]{x}$

23) $f(x) = 8x^4(2x^2 - 9x + 1)$

25) $f(x) = \frac{7}{x^7} - \frac{2}{x^5} + \frac{3}{x^2}$

<u>Part VII:</u> Use the Product Rule to find the derivative of the given functions. **DO NOT FOIL THE EXPRESSION**! Leave your answer in the unsimplified form.

26) $f(x) = (9x^3 + 7x - 4)(2x^5 - 8x + 1)$

27) $f(x) = (8x^2 - 6x + 1)(-3x^4 + 9x - 2)$

<u>Part VIII:</u> Use the Quotient Rule to find the derivative of the given functions. Simplify your function as far as possible.

28) $f(x) = \frac{8x-2}{9x+1}$

29) $f(x) = \frac{2x^2-5}{9x^3+1}$

Part IX: Use your calculator to find the value of the derivative of the given function at the given point

30) $f(x) = 4e^{7x^2-2}; f'(0.3)$

31) $f(x) = -6cos^4(2x); f'(-1)$

Part X: Write the equation of the line (a) tangent to the function and (b) normal to the function at the given point

32) $f(x) = 9x^3 + 2x;\ (1, 11)$

33) $f(x) = \sqrt[3]{x} - 2x;\ (8, -14)$

Part XI: Use the Chain Rule to find the derivative of the given function

34) $f(x) = (3x^5 - 2)^4$

35) $f(x) = \sqrt[3]{7x^9 + 3x}$

Part XII: Find the 2nd derivative of the given function

36) $f(x) = 9x^6 - 4x^3 + 8x - 1$

37) $f(x) = 2\sqrt[3]{x^5} - 4\sqrt[5]{x^8} + \frac{4}{x^5}$

Part XIII: Use Implicit Differentiation to find dy/dx

38) $3x^2 - 5y^2 = 10x$

39) $-6x^4 + 2x^2y^6 = 9y^2 - 3$

Part XIV: Use Implicit Differentiation to find d^2y/dx^2

40) $-2x^2 + 7y^2 = 20$

Part XV: Find the derivative of each exponential function

41) $f(x) = 8e^{9x^2-4}$

42) $f(x) = 9^{x^2-1}$

43) $f(x) = 8x^5 e^{\cos x}$

44) $f(x) = \frac{4x^7}{e^{8x^9}}$

Part XVI: Find the derivative of each logarithmic function

45) $f(x) = \ln(4x^5 - 8)$

46) $f(x) = \log_3(2x^6)$

47) $f(x) = 4x^5 \ln(8x^7)$

48) $f(x) = \frac{4x^4}{\ln(9x^3)}$

Part XVII: Find the derivative of each trigonometric function

49) $f(x) = \sin(e^{6x})$

50) $f(x) = 4\csc(\ln x)$

51) $f(x) = 8x^6 \tan(2x^9)$

52) $f(x) = \frac{6x^8}{\sec(2x^4)}$

Part XVIII: Find the derivative of each inverse trigonometric function

53) $f(x) = \cos^{-1}(e^{4x})$

54) $f(x) = \tan^{-1}(2x^3)$

55) $f(x) = \csc^{-1}(4x^8)$

56) $f(x) = \sec^{-1}(\sin x)$

Part XIX: $g(x)$ is the inverse of $f(x)$. Use the table to find the indicated derivative values for $g(x)$

x	1	2	3	4	5
$f(x)$	7	5	2	3	4
$f'(x)$	-1	4	-3	-6	2

57) $g'(3)$

58) $g'(5)$

Part XX: If $g(x) = f^{-1}(x)$, find the indicated $g'(x)$ values

59) $f(x) = -3x^4 + 2x + 9; g'(4)$

60) $f(x) = 2x^7 - 5x^2 + 2; g'(-1)$

Unit 4 Summer packet

1.

A particle moves along the *x*-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time *t* is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = 7/2$ only

(D) $t = 3$ and $t = 7/2$

(E) $t = 3$ and $t = 4$

2. A particle moves along a straight line with velocity given by $v(t) = 5 + e^{t/3}$ for time $t \geq 0$. What is the acceleration of the particle at time $t = 4$?

(A) 0.422

(B) 0.698

(C) 1.265

(D) 8.794

(E) 28.381

3. A particle moves along a line so that at time *t*, where $0 \leq t \leq \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity ofthe particle when its acceleration is zero?

(A) -5.19

(B) 0.74

(C) 1.32

(D) 2.55

(E) 8.13

4.

The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of *t* is the speed of the particle increasing?

(A) $3 < t < 4$ only

(B) $t > 4$ only

(C) $t > 5$ only

(D) $0 < t < 3$ and $t > 5$

(E) $3 < t < 4$ and $t > 5$

5. A tube is being stretched while maintaining its cylindrical shape. The height is increasing at the rate of 2 millimeters per second. At the instant that the radius of the tube is 6 millimeters, the volume is increasing at the rate of 96π cubic millimeters per second. Which of the following statements about the surface area of the tube is true at this instant? (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$. The surface area $S$of a cylinder, not including the top and bottom of the cylinder, is $S = 2\pi r h$.)

Unit 4 Summer packet

(A) The surface area is increasing by 28π square millimeters per second.
(B) The surface area is decreasing by 28π square millimeters per second.
(C) The surface area is increasing by 32π square millimeters per second.
(D) The surface area is decreasing by 32π square millimeters per second.

6. A particle moves on the hyperbola $xy = 15$ for time $t \geq 0$ seconds. At a certain instant, $x = 3$ and $\frac{dx}{dt} = 6$. Which of the following is true about y at this instant?

(A) y is decreasing by 10 units per second.
(B) y is increasing by 10 units per second.
(C) y is decreasing by 5 units per second.
(D) y is increasing by 5 units per second.

7. A particle moves along the curve $y = \frac{15}{x^2+1.3^x}$ for $x > 0$. The x-coordinate of the particle changes at a constant rate of 3 units per second. At the instant when the y-coordinate of the particle is 2, what is the rate of change of the y-coordinate of the particle, in units per second?

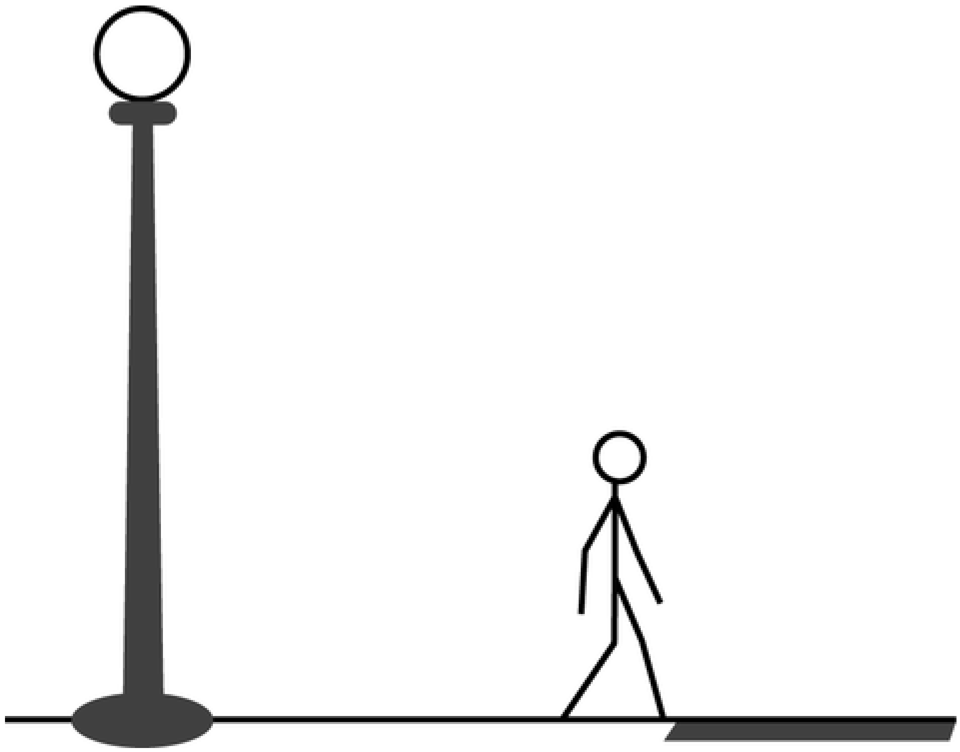
(A) -0.466
(B) -0.787
(C) -1.397
(D) -4.190

8. Paint spills onto a floor in a circular pattern. The radius of the spill increases at a constant rate of 2.5 inches per minute. How fast is the area of the spill increasing when the radius of the spill is 18 inches?

(A) $5\pi \text{ in}^2/\text{min}$
(B) $36\pi \text{ in}^2/\text{min}$
(C) $45\pi \text{ in}^2/\text{min}$
(D) $90\pi \text{ in}^2/\text{min}$

Unit 4 Summer packet

9.



A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec

(B) 2.667 ft/sec

(C) 3.75 ft/sec

(D) 6 ft/sec

(E) 10 ft/sec

10. A sphere is expanding in such a way that the area of any circular cross section through the sphere's center is increasing at a constant rate of $2\,cm^2/$ sec. At the instant when the radius of the sphere is 4 centimeters, what is the rate of change of the sphere's volume? (The volume *V* of a sphere with radius *r* is given by $V = \frac{4}{3}\pi r^3$.)

(A) $8\,cm^3/$ sec

(B) $16\,cm^3/$ sec

(C) $8\pi\,cm^3/$ sec

(D) $64\pi\,cm^3/$ sec

(E) $128\pi\,cm^3/$ sec

11. A differentiable function *f* has the property that $f(5) = 3$ and $f'(5) = 4$. What is the estimate for $f(4.8)$ using the local linear approximation for *f* at x= 5 ?

(A) 2.2
(B) 2.8
(C) 3.4
(D) 3.8
(E) 4.6

12. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4
(B) 0.5
(C) 2.6
(D) 3.4
(E) 5.5

13. Let f be a differentiable function with $f(2)=3$ and $f'(2)=\frac{1}{2}$. Using the line tangent to the graph of f at $x=2$ as a local linear approximation for f, what is the estimate for $f(1.8)$?

(A) 2.5
(B) 2.8
(C) 2.9
(D) 3.1

14. Let g be a differentiable function such that $g(3)=2$ and $g'(3)=-\frac{3}{4}$. The graph of g is concave down on the interval $(2,4)$. Which of the following is true about the approximation for $g(2.6)$ found using the line tangent to the graph of g at $x=3$?

(A) $g(2.6)\approx 1.7$ and this approximation is an overestimate of the value of $g(2.6)$.
(B) $g(2.6)\approx 1.7$ and this approximation is an underestimate of the value of $g(2.6)$.
(C) $g(2.6)\approx 2.3$ and this approximation is an overestimate of the value of $g(2.6)$.
(D) $g(2.6)\approx 2.3$ and this approximation is an underestimate of the value of $g(2.6)$.

15.

x	2.8	3.0	3.2	3.4
$g'(x)$	1.05	-1.2	-0.8	1.3

Selected values of the derivative of the function g are given in the table above. It is known that $g(3)=17$. What is the approximation for $g(3.2)$ found using the line tangent to the graph of g at $x=3$?

(A) 16.76
(B) 16.80
(C) 16.84
(D) 17.40

Unit 4 Summer packet

16. $\lim\limits_{x \to 0} \frac{x^2}{1-\cos x}$ is

(A) -2

(B) 0

(C) 1

(D) 2

(E) nonexistent

17. $\lim\limits_{x \to \infty} \frac{x^3}{e^{3x}}$ is

(A) 0

(B) $\frac{2}{9}$

(C) $\frac{2}{3}$

(D) 1

(E) infinite

18. $\lim\limits_{x \to \infty} \frac{\ln(e^{3x}+x)}{x} =$

(A) 0

(B) 1

(C) 3

(D) ∞

19. $\lim\limits_{t \to 0} \frac{\sin t}{\ln(2e^t-1)} =$

(A) -1

(B) 0

(C) $\frac{1}{2}$

(D) 1

20. $\lim\limits_{x \to \frac{\pi}{2}} \frac{3\cos x}{2x-\pi}$ is

(A) $-\frac{3}{2}$

(B) 0

(C) $\frac{3}{2}$

(D) nonexistent

Unit 5 Summer Packet

1. A differentiable function f has the property that $f'(x) \leq 3$ for $1 \leq x \leq 8$ and $f(5) = 6$. Which of the following could be true?

I. $f(2) = 0$

II. $f(6) = -2$

III. $f(7) = 13$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

2. The function f is given by $f(x) = x^3$. The application of the Mean Value Theorem to f on the interval $[1, 3]$ guarantees a point in the interval $(1, 3)$ at which the slope of the line tangent to the graph of f is equal to which of the following?

(A) 0
(B) 12
(C) 13
(D) 24

3. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

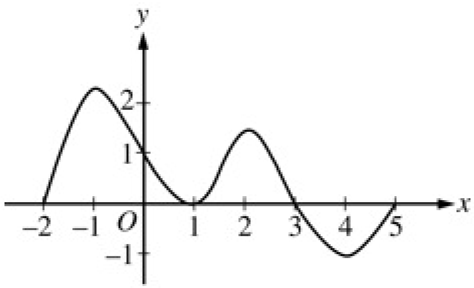
(A) -3
(B) $-\frac{3}{2}$
(C) $-\frac{1}{3}$
(D) 0
(E) There is no such k.

4. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval (0,10) ?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

Unit 5 Summer Packet

5.



Graph of f'

The graph of f' the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2,1]$ only

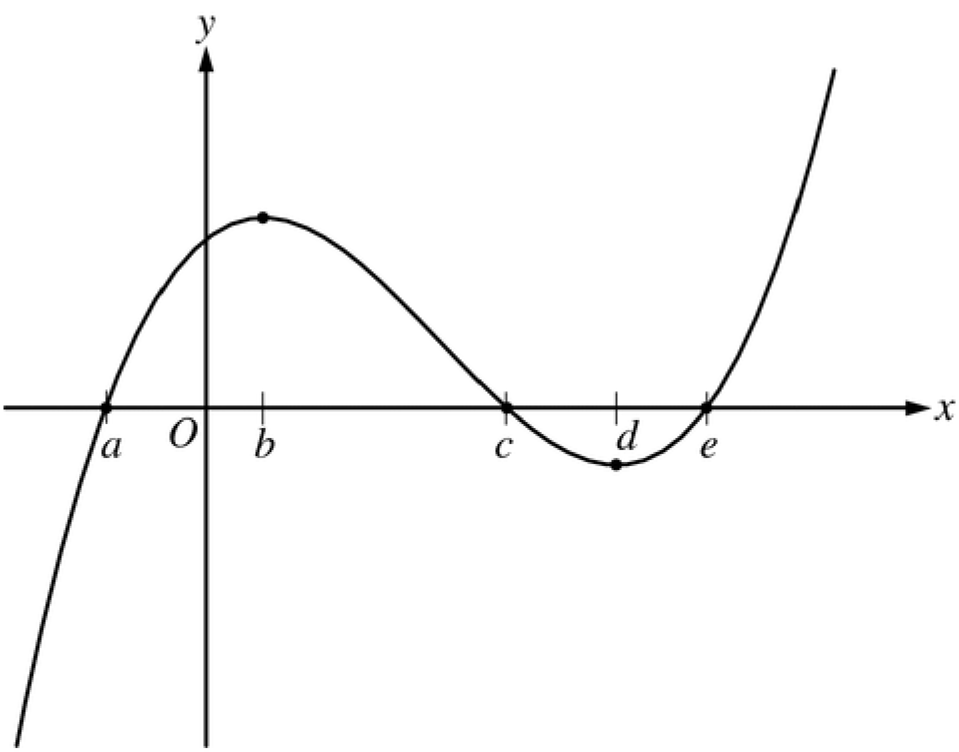
(B) $[-2,3]$

(C) $[3,5]$ only

(D) $[0,1.5]$ and $[3,5]$

(E) $[-2,-1]$, $[1,2]$ and $[4,5]$

6.



Graph of f

The figure above shows the graph of the polynomial function f. For which value of x is it true that $f''(x) < f'(x) < f(x)$?

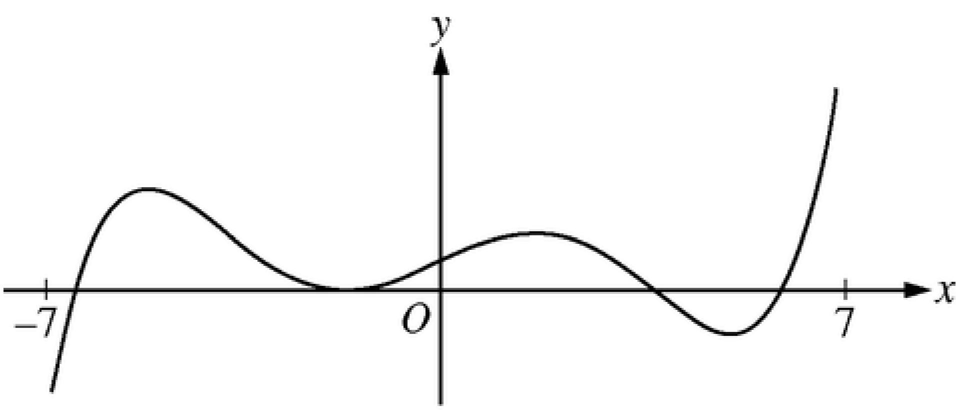
Unit 5 Summer Packet

(A) a
(B) b
(C) c
(D) d
(E) e

7. Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

8.



Graph of f'

The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Five

9. What is the absolute minimum value of $y = -\cos x - \sin x$ on the closed interval $\left[0, \frac{\pi}{2}\right]$?

(A) $-2\sqrt{2}$
(B) -2
(C) $-\sqrt{2}$
(D) -1

Unit 5 Summer Packet

10. For $-1.5 < x < 1.5$, let *f* be a function with first derivative given by $f'(x) = e^{(x^4-2x^2+1)} - 2$. Which of the following are all intervals on which the graph of *f* is concave down?

(A) (-0.418, 0.418) only

(B) (-1, 1)

(C) (-1.354, -0.409) and (0.409, 1.354)

(D) (-1.5, -1) and (0,1)

(E) (-1.5, -1.354), (-0.409, 0), and (1.354, 1.5)

11. Let *f* be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of *x* at which the graph of *f* changes concavity?

(A) 0.56

(B) 0.93

(C) 1.18

(D) 2.38

(E) 2.44

12.

x	0	1	2	3	4	5
$f'(x)$	-3	0	-1	5	0	-3
$f''(x)$	5.3	-2.0	1.7	-0.5	1.2	-5.1

Let f be a twice-differentiable function. Selected values of f' and f'' are shown in the table above. Which of the following statements are true?

I. f has neither a relative minimum nor a relative maximum at $x = 1$.
II. f has a relative maximum at $x = 1$.
III. f has a relative maximum at $x = 4$.

(A) I only

(B) II only

(C) III only

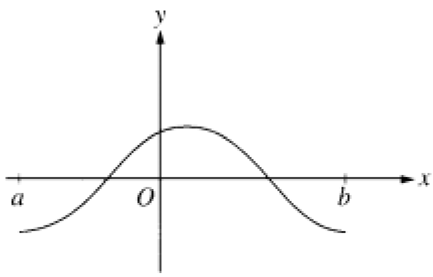
(D) I and III only

13. Let f be a twice-differentiable function. Which of the following statements are individually sufficient to conclude that $x = 4$ is the location of the absolute maximum of f on the interval $[0, 10]$?

I. $f'(4) = 0$
II. $x = 4$ is the only critical point of f on the interval $[0, 10]$, and $f''(4) < 0$.
III. $x = 4$ is the only critical point of f on the interval $[0, 10]$, and $f(10) < f(0) < f(4)$.

(A) II only
(B) III only
(C) I and II only
(D) II and III only

14.



The graph of f is shown in the figure above. Which of the following could be the graph of derivative of f ?

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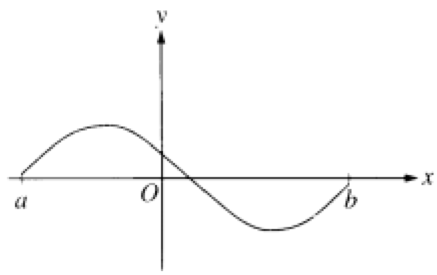
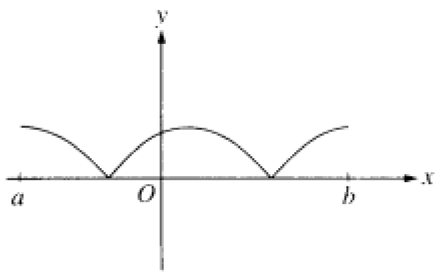
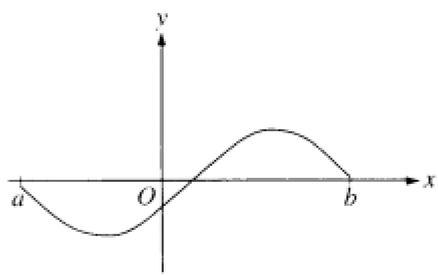
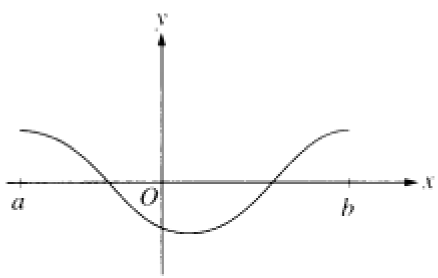
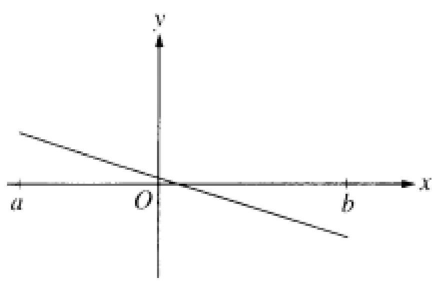
(A) 

(B) 

(C) 

(D) 

(E) 

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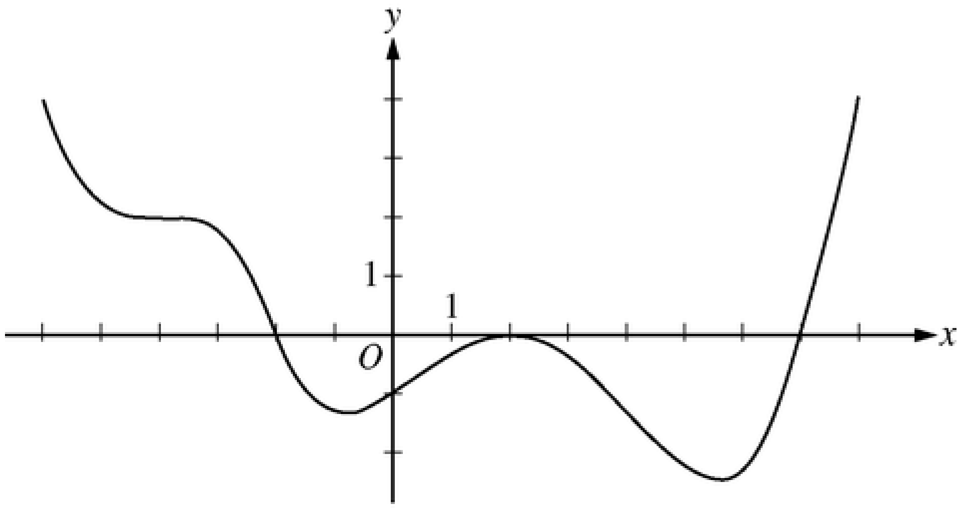
15.



Graph of f'

The figure above shows the graph of f', the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection

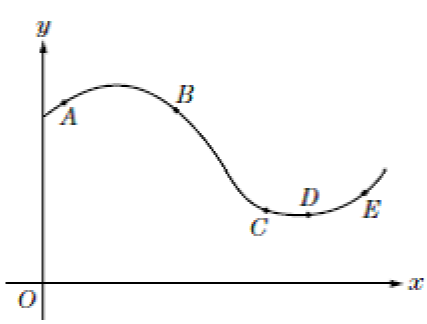
(B) 1 relative minimum, 1 relative maximum, and 4 points of inflection

(C) 2 relative minima, 1 relative maximum, and 2 points of inflection

(D) 2 relative minima, 1 relative maximum, and 4 points of inflection

(E) 2 relative minima, 2 relative maxima, and 3 points of inflection

16. At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?



(A) A

(B) B

(C) C

(D) D

(E) E

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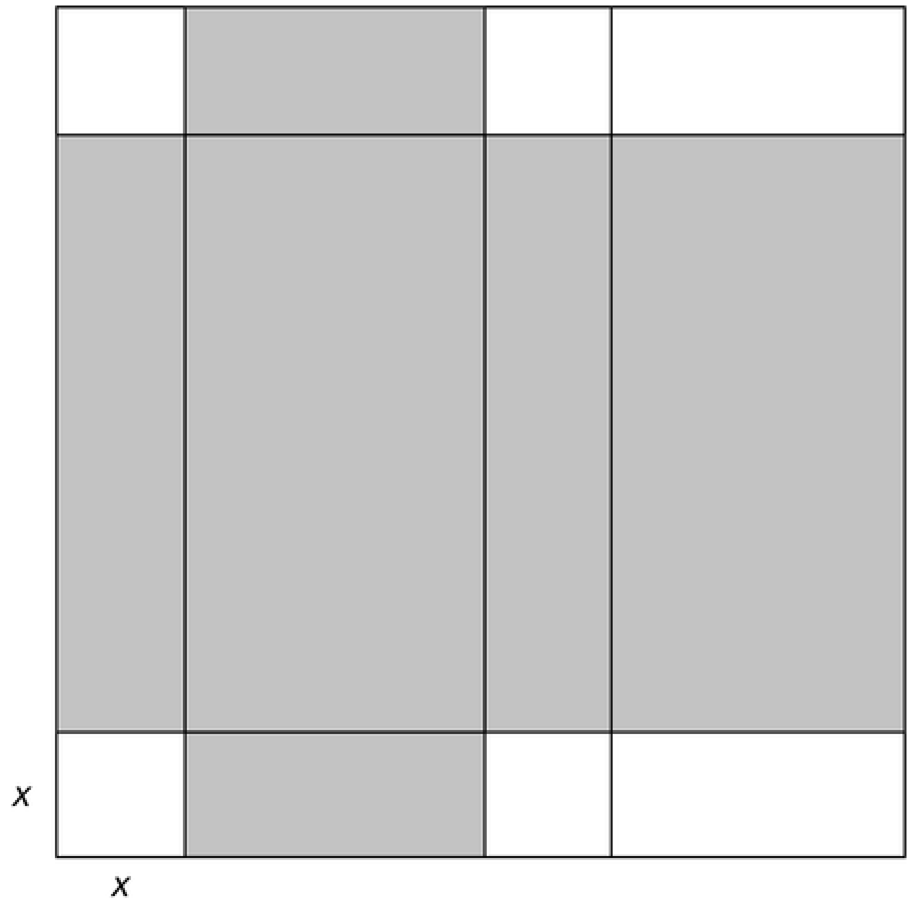
17.



The figure above represents a square sheet of cardboard with side length 20 inches. The sheet is cut and pieces are discarded. When the cardboard is folded, it becomes a rectangular box with a lid. The pattern for the rectangular box with a lid is shaded in the figure. Four squares with side length x and two rectangular regions are discarded from the cardboard. Which of the following statements is true? (The volume V of a rectangular box is given by $V = lwh$.)

(A) When $x = 10$ inches, the box has a minimum possible volume.

(B) When $x = 10$ inches, the box has a maximum possible volume.

(C) When $x = \frac{10}{3}$ inches, the box has a minimum possible volume.

(D) When $x = \frac{10}{3}$ inches, the box has a maximum possible volume.

18. Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm

(B) 10 cm

(C) 20 cm

(D) $\frac{30}{\pi^2}$ cm

(E) $\frac{10}{\pi}$ cm

Unit 5 Summer Packet

19. The speed of a runner, in miles per hour, on a straight trail is modeled by $f(m) = \frac{1}{10}\left(-2m^3 + 9m^2 - 12m\right) + 7$, where m is the runner's distance, in miles, from the start of the trail. What is the maximum speed of the runner for $0 \le m \le 3$?

(A) 6.5

(B) 6.6

(C) 7.0

(D) 7.5

20. A rectangular area is to be enclosed by a wall on one side and fencing on the other three sides. If 18 meters of fencing are used, what is the maximum area that can be enclosed?

(A) $\frac{9}{2}\,m^2$

(B) $\frac{81}{4}\,m^2$

(C) $27\,m^2$

(D) $40\,m^2$

(E) $\frac{81}{2}\,m^2$